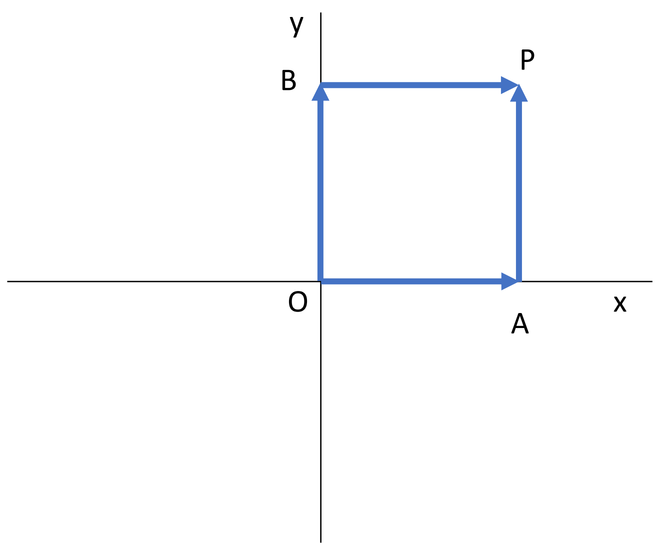
Solution 23

1) Given $\vec{F} = -xy\hat{x} + xy\hat{y}$
Conservative?

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial \mathrm{y}} & \dfrac{\partial}{\partial \mathrm{z}} \\ \text{-xy} & \text{xy} & 0 \end{vmatrix} = \left(\frac{\partial(xy)}{\partial x} - \frac{\partial(-xy)}{\partial y}\right)\hat{z} = (y + x)\hat{z}$$

This is not zero everywhere. Hence this is a non-conservative force. Consequently, the work done over a path does not only depend on the start and end point but -in general- also on the path itself.

2)



Path OAP:

$$W_{OAP} = \int_{x=0,y=0}^{x=1,y=0} F_x dx + \int_{x=1,y=0}^{x=1,y=1} F_y dy = 0\int_0^1 \text{-}x dx + 1\int_0^1 \mathrm{y} dy = \frac{1}{2}$$

Path OBP:

$$W_{OBP} = \int_{x=0,y=0}^{x=0,y=1} F_y dy + \int_{x=0,y=1}^{x=1,y=1} F_x dx = 0\int_0^1 \mathrm{y} dy + 1\int_0^1 \text{-xdx} = \text{-}\frac{1}{2}$$

Indeed, the amount of work turns out to be path dependent. We could expect this based on (1). However, if a force is not conservative it does not mean that the work done when going from point 1 to point 2 will always be different for different paths. It may very well be that the amount of work along some paths (or even many) will be the same.

It does mean that there are paths that give different outcomes. Or rephased: if a force is not-conservative then “not for every closed path is the amount of work done zero”.