

Exercise non-conservative forces

$$\text{a) } \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & 0 \end{vmatrix} = 0\hat{x} - 0\hat{y} + \left(\frac{\partial}{\partial x} - x - \frac{\partial}{\partial y} y \right) \hat{z} = -2\hat{z}$$

thus F is not conservative.

b) Stokes' Theorem:

$$\oint \vec{F} \cdot d\vec{r} = \iint \vec{\nabla} \times \vec{F} \cdot d\vec{\sigma} =$$

we have: $\vec{\nabla} \times \vec{F} = -2\hat{z}$ and $d\vec{\sigma} = +d\sigma\hat{z}$
the latter is due to the anti-clockwise direction we 'walk the circle'.

Hence,

$$W = \oint \vec{F} \cdot d\vec{r} = \iint \vec{\nabla} \times \vec{F} \cdot d\vec{\sigma} = -2 \underbrace{\hat{z} \cdot \hat{z}}_{=1} \underbrace{\iint d\sigma}_{\pi 1^2} = -2\pi$$

c) Obviously, the work done is negative. This means that the kinetic energy of the object that is moved around is decreasing. The force is thus acting against the motion. You can easily check this for yourself by drawing at a few positions on the unit circle the direction of the force.

